Lumped Device Modeling with FDTD including Packaging Effects

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Abstract — **A two-terminal device can be incorporated into FDTD using the Lumped Element FDTD (LE-FDTD) formulation, unfortunately, this method is not able to analyze accurately the packaging effects of the device. This is possible using Lumped Network FDTD (LN-FDTD) - however LN-FDTD requires a complicated pre-calculation for simple devices such as Schottky or varactor diode. Therefore, this paper presents a simple and effective state space approach to incorporate device packaging effects into FDTD algorithm. The new technique is validated by means of experimental measurements of a varactor-tuned patch antenna and the agreement between the predicted and actual responses is shown to be excellent.**

I. INTRODUCTION

The Finite-Difference-Time-Domain (FDTD) approach is well-known and widely-used for the full wave electromagnetic analysis of three-dimensional structures.

A simple two-terminal device such as a varactor may be included in the FDTD analysis by means of Lumped Element FDTD (LE-FDTD) [1]-[2]. However, the conventional LE-FDTD method cannot include the packaging effects of the device, such as lead inductance, package capacitance and chip resistance. These play an important part in the behaviour of the component.

Such packaging effects may be incorporated by means of the Lumped Network FDTD (LN-FDTD) formulation [3]-[4]. However, in this contribution, a new and simpler technique is demonstrated which combines the LE-FDTD method with a State Variable formulation. This new method does not require a bilinear transformation (Laplace domain to z-domain), which essentially removes the pre-computational complication to the method.

One application of a varactor diode is for tuning the resonant frequency of a microstrip patch antenna, which was first reported by [5]. This method of tuning allows the inherently narrowband patch antenna to cover a very large bandwidth (typically in excess of 30%) in discrete narrowband intervals. The modelling technique proposed herein is demonstrated by application to a varactor-tuned patch antenna.

II. DEVICE MODELLING IN FDTD

A. Conventional LE-FDTD

Consider the Maxwell's curl H equation in free space, appropriate for updating the electric field:

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad (1)$$

The LE-FDTD method includes a lumped circuit component by means of a lumped element current density $\mathbf{J}_L$:

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_L \quad (2)$$

$$\mathbf{E}^{n+1} = \mathbf{E}^n + \frac{\Delta t}{\varepsilon_0}(\nabla \times \mathbf{H})^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0}\mathbf{J}_L^{\,n+\frac{1}{2}} \quad (3)$$

B. Extended Methodology

For the lumped current density to represent an entire lumped network, there are three main steps to be followed:

1) Describe the network circuit using the State Variable formulation in the Laplace domain.
2) Transform the State Variable formulation from the Laplace domain to the time domain using the Trapezoidal formula.
3) Representing the lumped current density by utilizing the current through the input.

Firstly, the state variables are identified with the energy storage elements in the circuit (current in the case of an inductor, voltage for a capacitor). Solving the network circuit using loop analysis eventually yields the state variable form. The network is then described by the following system of simple, first order equations:

$$s\mathbf{CX} = -\mathbf{GX} + \mathbf{W} \quad (4)$$

Here, $\mathbf{X}$ is the state variable vector (a combination of voltages and currents in the circuit) and $\mathbf{W}$ is the input vector, in this case, the potential difference across the

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device. **C** represents the capacitance or inductance and **G** represents the resistance or conductance used in the circuit.

In order to couple the state variable formulation and FDTD algorithm, a transformation of (4) from the Laplace domain to the time domain is essential - if n is the current iteration and the prime indicates differentiation with respect to time:

$$\mathbf{C}\mathbf{x}'_n = -\mathbf{G}\mathbf{x}_n + \mathbf{w}_n \tag{5}$$

To calculate $\mathbf{x}'_n$ the causal Forward Euler formulation or the non-causal Backward Euler and Trapezoidal formulation may be used. Although the Trapezoidal formulation yields second-order accuracy, it is difficult to couple into the FDTD as it requires prior knowledge of next step electric field, $\mathbf{w}_{n+1}$. As a result, to predict the future information needed in the Trapezoidal formulation, the Forward Euler formulation has to be used as a *predictor*. Later, the predicted information will be passed to the *corrector*, which uses the Trapezoidal formulation:

So, employing the Forward Euler form to approximate the derivative:

$$\mathbf{x}'_n = \frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{\Delta t} \tag{6}$$

yields:

$$\mathbf{C}\mathbf{x}_{n+1} = (\mathbf{C} - \Delta t\mathbf{G})\mathbf{x}_n + \Delta t\mathbf{w}_n \tag{7}$$

One of the terms in $\mathbf{x}_{n+1}$ will be the current supplied to the whole network, which can be used to update the electric field in (3). A semi-implicit method [2] is used to discretize the current term **J** at n+1/2. The result will be the predicted next step electric field, which is $\mathbf{w}_{n+1}$:

$$\mathbf{E}^{n+1}\Big|_{PRE} = \mathbf{E}^n + \frac{\Delta t}{\varepsilon_0}(\nabla\times\mathbf{H})^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0}\left(\frac{\mathbf{J}_L^{\;n+1} + \mathbf{J}_L^{\;n}}{2}\right) \tag{8}$$

Now using the Trapezoidal form:

$$\frac{\mathbf{x}'_{n+1} + \mathbf{x}'_n}{2} = \frac{\mathbf{x}_{n+1} - \mathbf{x}_n}{\Delta t} \tag{9}$$

(5) then yields:

$$\left(\mathbf{C} + \frac{\Delta t}{2}\mathbf{G}\right)\mathbf{x}_{n+1} = \left(\mathbf{C} - \frac{\Delta t}{2}\mathbf{G}\right)\mathbf{x}_n + \frac{\Delta t}{2}(\mathbf{w}_{n+1} + \mathbf{w}_n) \tag{10}$$

If the network is linear, this set of simultaneous equations may be solved by any standard technique. If the network is non-linear, the simultaneous Newton-Raphson Iteration method may be used.

Again, one of the terms in $\mathbf{x}_{n+1}$ is the current supplied to the whole network, which is used to correct the previous electric field to give a 2[nd]-order accurate result:

$$\mathbf{E}^{n+1}\Big|_{COR} = \mathbf{E}^n + \frac{\Delta t}{\varepsilon_0}(\nabla\times\mathbf{H})^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon_0}\left(\frac{\mathbf{J}_L^{\;n+1} + \mathbf{J}_L^{\;n}}{2}\right) \tag{11}$$

Normally, the time step Δt chosen to satisfy Courant condition is smaller than the time step required for the microwave network circuit. Thus the state variable formulation is stable.

III. PACKAGED DEVICE MODEL

The equivalent circuit diagram of a varactor, including the parasitic components associated with the package, is shown in Fig. 1.

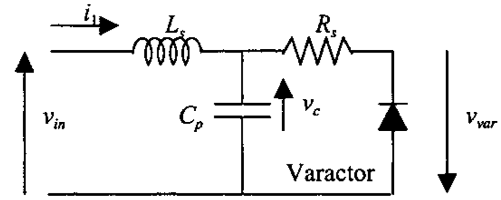

Fig. 1. Varactor equivalent circuit, including packaging

The junction capacitance C_j of the varactor exhibits a nonlinearity according to [6]:

$$C_j(v_{var}) = \frac{C_{j0}}{\left(1 - \frac{V_R + v_{var}}{\phi}\right)^{\gamma}} \tag{12}$$

- v_{var} is the voltage across the varactor, C_{j0} is the static capacitance, V_R is the applied reverse bias, γ is the exponent of the C-V relation and ϕ is the built-in potential.

To include the packaging effects, the state variable formulation described in the previous section is employed. The varactor is orientated in the z-direction and input voltage (v_{in}) is $\Delta z E_z$. Three state variables x are identified with the energy storage in the lead inductance (L_s), package capacitance (C_p) and junction capacitance (C_j):

$$x_1 = i_1 \tag{13a}$$

$$x_2 = v_c \tag{13b}$$

$$x_3 = v_{var} \tag{13c}$$

TABLE 1

COMPARISONS OF MEASURED AND PREDICTED RESONANT FREQUENCIES

Reverse Bias Voltage (V)	Measured (MHz)	Predicted (MHz)	Error (%)
0	1661	1659	0.12
-5	1761	1756	0.28
-30	1784	1781	0.17
No Varactor	1805	1805	0.00

Using simple circuit analysis, the relationships between these state variables can be shown as:

$$s\begin{bmatrix} L_s & 0 & 0 \\ 0 & C_p & 0 \\ 0 & 0 & C_j(v_{var}) \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1/R_s & -1/R_s \\ 0 & -1/R_s & -1/R_s \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} v_{in} \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

Using (7), the Forward Euler formulation yields a prediction $\mathbf{x}^{n+1}$

$$\begin{bmatrix} L_s & 0 & 0 \\ 0 & C_p & 0 \\ 0 & 0 & C_j(x_3^n) \end{bmatrix}\begin{bmatrix} x_1^{n+1} \\ x_2^{n+1} \\ x_3^{n+1} \end{bmatrix} = \begin{bmatrix} L_s & \Delta t & 0 \\ -\Delta t & C_p - \Delta t/R_s & -\Delta t/R_s \\ 0 & -\Delta t/R_s & C_j(x_3^n) - \Delta t/R_s \end{bmatrix}\begin{bmatrix} x_1^n \\ x_2^n \\ x_3^n \end{bmatrix} + \Delta t\begin{bmatrix} \Delta z E_z^n \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

Using (10), the Trapezoidal formulation gives a *corrector* for $\mathbf{x}^{n+1}$:

$$\begin{bmatrix} L_s & \Delta t/2 & 0 \\ -\Delta t/2 & C_p + \Delta t/2R_s & \Delta t/2R_s \\ 0 & \Delta t/2R_s & C_j\left(\left(x_3^{n+1} + x_3^n\right)/2\right) + \Delta t/2R_s \end{bmatrix}\begin{bmatrix} x_1^{n+1} \\ x_2^{n+1} \\ x_3^{n+1} \end{bmatrix} = \begin{bmatrix} L_s & -\Delta t/2 & 0 \\ \Delta t/2 & C_p - \Delta t/2R_s & -\Delta t/2R_s \\ 0 & -\Delta t/2R_s & C_j\left(\left(x_3^{n+1} + x_3^n\right)/2\right) - \Delta t/2R_s \end{bmatrix}\begin{bmatrix} x_1^n \\ x_2^n \\ x_3^n \end{bmatrix} + \frac{\Delta t \Delta z}{2}\begin{bmatrix} E_z^{n+1}\big|_{PRE} + E_z^n \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

(8) being used to find $E_z^{n+1}\big|_{PRE}$. The Newton-Raphson numerical method is used to solve the implicit relationship for x_3 in (16). $x_1 = i_1$ represents the current through the whole packaged device and thus this may be inserted into FDTD in the place of $\mathbf{J}_L$ ($J_{Lz} = i_1/\Delta x \Delta y$) in equation (11) using the semi-implicit method.

Experience with this algorithm has shown that it is stable with a time step given by the standard Courant condition for the FDTD method.

IV. RESULT AND DISCUSSION

A varactor-tuned patch antenna was constructed. The dimensions of the patch were 26×17mm on RT/Duroid 6010 substrate of permittivity ε_r=10.2 and a thickness of 0.8mm. Tuning was accomplished by means of a silicon abrupt junction varactor (SMV1405-079 from Alpha Industries) - from the manufacturer's data sheet, C_{j0}=2.57pF, γ=0.39, φ=0.68 for the varactor and L_s=0.7nH, C_p=0.05pF, R_s=0.8Ω for the package. The patch was pin-fed and the varactor was connected between the edge of the patch and ground, by means of a via shown in Fig. 2.

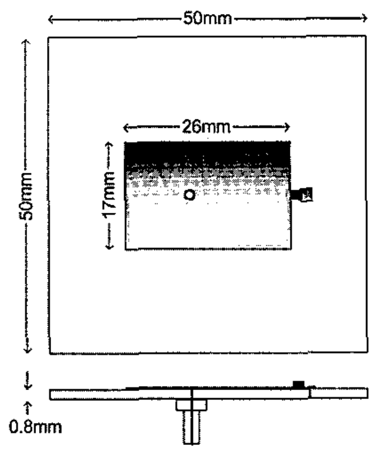

Fig 2. Varactor-tuned patch antenna

As shown in Table 1, over a range of reverse bias voltages from 0 to 30V, the method presented in section 2 is able to track the operating frequency of the patch with a typical accuracy of 5MHz (0.28%). Furthermore, the shape of the measured return loss in Fig. 3 is closely matched over the full range of bias conditions and over the full band of interest.

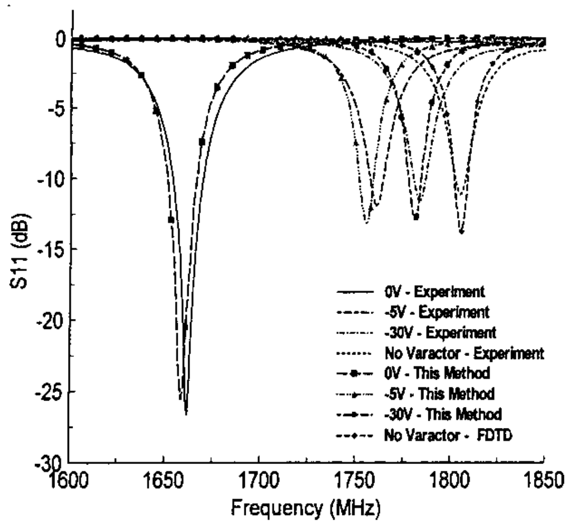

Fig 3. Measured and Predicted S11 of the microstrip tuned patch antenna.

Importantly, if the packaging effects are neglected the discrepancy between FDTD and measurement increases considerably, for example an error of 35MHz (2.0%) between the measured and non-packaged varactor occurs at 0V bias. This demonstrates the importance of including the package effects in the FDTD model.

V. CONCLUSIONS

A simple technique that extends the original LE-FDTD method to include packaging effects of a device has been presented. The technique introduces minimal computational overheads, and has no observed effect on algorithm stability.

This new method has been validated by considering a varactor-tuned patch antenna. The numerical and experimental results for this antenna are shown to agree with pleasing accuracy over a wide range of bias voltages.

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